Outside Options in Neutral Allocation of Discrete Resources*

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Abstract

Serial dictatorships have emerged as natural direct mechanisms in the literature on the allocation of indivisible goods without transfers. They are the only neutral and group–strategy-proof mechanisms in environments in which agents have no outside options and hence no individual rationality constraints (Svensson, 1999). Accounting for outside options and individual rationality constraints, our main result constructs the class of group–strategy-proof, neutral, and non-wasteful mechanisms. These mechanisms are also Pareto efficient and we call them binary serial dictatorships. The abundance of the outside option—anybody who wants can opt out to get it—is crucial for our result.

1 Introduction

Serial dictatorships have often emerged as natural direct mechanisms in the literature on the allocation of indivisible objects without transfers and with single-unit demands (i.e., the Hylland and Zeckhauser, 1979 model). A serial dictatorship mechanism allocates objects by ordering agents, and then letting the first agent choose her most preferred object, thereafter letting the second agent choose his most preferred object among those still available, etc. Svensson (1999) explains the attractiveness of serial dictatorships by showing that they are the only neutral and group strategy-proof mechanisms. A mechanism is neutral if its outcome does not depend on the labelling of objects.[1] A mechanism is group strategy-proof if there is no group of agents that can misstate their preferences and obtain a weakly better house, and such that at least one agent in the group gets a strictly better house. Svensson restricts attention to environments in which agents have no outside options and hence no individual rationality constraints.

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[1]Neutrality allows the mechanism to depend on how the outside options are called; this plays no role in Svensson's setting but matters in the environments with outside options that we study.

We allow for the outside options: each agent can remain unmatched if she chooses to, i.e., participation is voluntary. Our main result establishes that the class of group strategy-proof, neutral, non-wasteful and individually-rational mechanisms consists of mechanisms we call binary serial dictatorships. Individual rationality ensures voluntary participation: no agent is assigned a house worse than her outside option. Non-wastefulness is a weak efficiency property: a mechanism is non-wasteful if there is no unassigned house that an agent prefers to be matched with rather than her assignment. The class of binary serial dictatorships generalizes serial dictatorships to the setting with outside options. A binary serial dictatorships first assigns a selected agent her most preferred outcome among all houses and her outside option; we also refer to being assigned the outside option as being unmatched. A second agent is then assigned his most preferred outcome among all not-yet-assigned houses and his outside option. In contrast to serial dictatorships, the identity of the second agent can depend on whether the first agent is matched with a house or with an outside option. The mechanism then repeats the procedure, selecting a third agent whose identity depends on whether the first and second agent were matched with houses or outside options, etc.[2]

Our characterization has two corollaries. First, because binary serial dictatorships are Pareto efficient, we can conclude that binary serial dictatorships are also the class of group strategy-proof, neutral, Pareto efficient and individually-rational mechanisms. Second, in the subdomain of our preference domain in which the outside option is always ranked last by all agents – the domain that most previous axiomatic studies on house allocation used –, our result implies that a mechanism is group strategy-proof, neutral, and non-wasteful if and only if it is a serial dictatorship. As an auxiliary result, we extend to the environment with outside options Pápai's (2001) insight that group strategy-proofness is equivalent to individual strategy-proofness and non-bossiness.

The present paper is the first to analyze the voluntary participation in allocation of indivisible goods without transfers in the presence of outside options. Serial dictatorships were introduced by Satterthwaite and Sonnenschein (1981) in private good economies and studied by Svensson (1994) in the house allocation context as a strategy-proof mechanism in absence of outside options (also see Roth, 1982). In addition to Svensson (1999), Ergin (2000) characterized serial dictatorships by maintaining the neutrality requirement and replacing group strategy-proofness with monotonicity and consistency axioms. Abdulkadiroğlu and Sönmez (1998) showed that, given a fixed preference profile, each Pareto efficient outcome can be obtained by running a serial dictatorship.[3] Sönmez and Ünver (2010) studied neutrality and strategy-proofness, together with additional axioms, and allow agents to have property rights over some of the goods (see also Abdulkadiroğlu and Sönmez, 1999 for this model). Pycia and Ünver (2020) showed that Arrovian efficient and strategy-proof mechanisms

[2]Surprisingly, the simple and elegant proof of Svensson hinges on the lack of outside options and does not extend to our environment; in effect, our argument is substantially more involved. See the discussion in Section 7.

[3]Abdulkadiroğlu and Sönmez also show that randomizing over serial dictatorships is equivalent to randomizing over Gale's top trading cycles (cf. Shapley and Scarf 1974, Ma 1994). For further studies of random serial dictatorships see Sönmez and Ünver (2005), Pathak and Sethuraman (2011), Che and Kojima (2010), Liu and Pycia (2011), Carroll (2014), and Pycia and Troyan (2019). For a related result for serial dictatorships, see Pycia (2019). For studies of dictatorships, see e.g. Gibbard (1973), Satterthwaite (1975), Hylland (1980), and Bahel and Sprumont (2020).

resemble sequential dictatorships except that in the last step of the algorithm, when there are only two goods left, two agents might be endowed with these goods and allowed to trade them; Pycia and Troyan (2016) showed that a similar class of sequential-dictatorship-like mechanisms characterizes strong obvious strategy-proofness and Pareto efficiency. These papers also did not allow agents to take outside options.

Following our work, others have examined outside options in related environments. Bu (2014) used neutrality and additional axioms to characterize sequential dictatorships. Erdil (2014) showed in a domain without transfers that non-wasteful and strategy-proof deterministic mechanisms are not dominated by strategy-proof deterministic mechanisms. In school-choice domain, Kesten and Kurino (2019) showed that with outside options there is no mechanism that Pareto-dominates the student-optimal stable school-choice mechanism. They also study maximal subdomains of preferences where such result no longer holds. In a more general setting with or without transfers, Alva and Manjunath (2019) showed that if a pair of individual rational and strategy-proof mechanisms are participation equivalent (i.e., if at every problem every agent either receives her outside option under both mechanisms or is assigned a non-outside-option outcome under both) then they should be welfare equivalent.[4] Calsamiglia, Martinez-Mora and Miralles (2020) showed that the presence of outside options has an even bigger impact on individually-rational but non-strategy-proof mechanisms as it enables agents with better outside option to choose more risky equilibrium strategies.

While we show that Svensson's serial dictatorship insight can be modified so as to make it valid when agents have outside options, there are many other standard mechanism design problems in which whether agents have the ability to take an outside option crucially affects the standard results. For instance, in the setting with monetary transfers and quasi-linear utilities, the impossibility of ex-post Pareto efficient and Bayesian incentive compatible bilateral trade shown by Myerson and Satterthwaite (1983) crucially depends on individual rationality. The Coasian dynamics of Gul, Sonneschein, and Wilson (1986) hinges on the inability of buyers to take an outside option, as shown by Board and Pycia (2014).

2 House Allocation Problem with Outside Options

Let I be a finite set of **agents**. Let H be a finite set of indivisible goods that we refer to as **houses** (following the terminology of Shapley and Scarf, 1974). Each agent i has a **strict preference relation** over H and her **outside option** denoted by $\emptyset$. The strict preference relation is denoted by $\succ_i$. Let $\succeq_i$ be the induced **weak preference relation** from $\succ_i$,[5] that is for any

[4] For other characterizations involving strategy-proofness in the house allocation domain see, for example, Pápai (2000), Ehlers (2002), Ehlers, Klaus, and Pápai (2002), Bogomolnaia, Deb, and Ehlers (2005), Kesten (2009), Bu (2014), Velez (2014), Pycia (2016), and Pycia and Ünver (2017). See Sönmez and Ünver (2011) for a recent survey of the literature. In the setting with multi-unit demand, Pápai (2001), Ehlers and Klaus (2003), and Hatfield (2009) characterized sequential dictatorships not allowing for outside options.

[5] The weak preference relation is a linear order on H, i.e. a binary relation on H that is antisymmetric, transitive, complete, and reflexive.

$x, y \in H \cup \{\emptyset\}$,

$$x \succeq_i y \quad \Longleftrightarrow \quad x = y \text{ or } x \succ_i y.$$

We denote the *preference relation* of agent i by the induced weak preference relation $\succeq_i$. Let $\mathcal{R}$ be the set of preference relations. Let $\succeq = (\succeq_i)_{i \in I} \in \mathcal{R}^{|I|}$ be a **preference profile**. Each agent has not only right to hold on to her own house, but also have rights on the vacant houses, which are social endowments. Triple $\langle I, H, \succeq \rangle$ is a **house allocation problem with outside options**.

An outcome of a problem is a *matching*. Following Pycia and Ünver (2017), we define the auxiliary concept of a *submatching* first. A **submatching** is an assignment that assigns a subset of agents a house or the remaining unmatched option, and no two agents the same house. Formally, for any given $J \subseteq I$, a submatching is a function $\sigma : J \longrightarrow H \cup \{\emptyset\}$ such that for any $i, j \in J$, $\sigma(i) = \sigma(j) \Rightarrow i = j$ or $\sigma(i) = \emptyset$. We will occasionally use the set interpretation of functions to denote the submatching σ as well, i.e., $\sigma = \{(i, \sigma(i))\}_{i \in J}$. Let $\mathcal{S}$ be the set of submatchings, which includes also the empty submatching $\varnothing$. We denote the set of agents over which the submatching σ is defined as $I^\sigma = J$; moreover, let H^σ be the houses matched in the submatching σ: $H^\sigma = \sigma(I^\sigma) \setminus \{\emptyset\}$. A **matching** is a submatching σ such that $I^\sigma = I$. Let $\mathcal{M}$ denote the set of matchings. Let $\overline{\mathcal{M}} = \mathcal{S} \setminus \mathcal{M}$ denote the set of submatchings that are not matchings..

A **mechanism** assigns a matching for each problem. Throughout the paper, we fix I and H, and thus, a problem is given by the preference profile $\succeq$. Therefore, formally a mechanism is a function $\varphi : \mathcal{R}^{|I|} \longrightarrow \mathcal{M}$.

2.1 Axioms

A matching is **individually rational**, if no agent receives a house worse than the outside option: $\mu \in \mathcal{M}$ is *individually rational* if for every $i \in I$, $\mu(i) \succeq_i \emptyset$. A mechanism is **individually rational**, if it always finds an individually-rational matching.

A matching is **non-wasteful**, if no agent receives an option that is worse than a house that is unassigned: $\mu \in \mathcal{M}$ is *non-wasteful* for every $i \in I$, $\mu(i) \succeq_i h$ for every $h \in H \setminus \mu(I)$. Non-wastefulness would imply individual rationality if $\emptyset$ were (equivalently) considered as a house with $|I|$ copies. Thus, one can think of individual rationality as a special instance of non-wastefulness.

A matching is **Pareto efficient**, if there is no matching that makes everybody weakly better off, and at least one agent strictly better off. That is, a matching $\mu \in \mathcal{M}$ is *Pareto efficient* if there exists no matching $\nu \in \mathcal{M}$ such that for every $i \in I$, $\nu(i) \succeq_i \mu(i)$, and for some $i \in I$, $\nu(i) \succ_i \mu(i)$. A mechanism is **Pareto efficient**, if it always finds a Pareto–efficient matching.

Individual rationality, non-wastefulness and Pareto efficiency are related concepts.

Lemma 1 *If a matching is Pareto efficient then it is individually rational and non-wasteful.*

Proof of Lemma 1. Let μ be an individually irrational or wasteful matching. Then there exists some agent $i \in I$, with $g \succ_i \mu(i)$ such that either $g = \emptyset$ or $g \in H$ is not assigned to any agent. Consider the following matching ν: for every $j \in I \setminus \{i\}$, $\nu(j) = \mu(j)$ and $\nu(i) = g$. Clearly ν Pareto-dominates μ; hence, μ is not Pareto efficient. ■

A mechanism is **non-bossy** if whenever an agent misreports her preferences and cannot change her house assigned by the mechanism, then she cannot change the matching assigned by the mechanism, either (Satterthwaite and Sonneschein, 1981). Formally, a mechanism φ is *non-bossy* if for every $\succeq \in \mathcal{R}^{|I|}$, $i \in I$, and $\succeq_i' \in \mathcal{R}$,

$$\varphi[\succeq_i', \succeq_{-i}](i) = \varphi[\succeq](i) \quad \Rightarrow \quad \varphi[\succeq_i', \succeq_{-i}] = \varphi[\succeq].$$

A mechanism is **strategy-proof** if an agent cannot receive a better house by misreporting her preferences. Formally, a mechanism φ is *strategy-proof* if for every $\succeq \in \mathcal{R}^{|I|}$, for every $i \in I$, and $\succeq_i' \in \mathcal{R}$,

$$\varphi[\succeq](i) \succeq_i \varphi[\succeq_i', \succeq_{-i}](i).$$

A mechanism is **group strategy-proof** if there is no group of agents that can misstate their preferences so that they all obtain a weakly better house and at least one agent in the group gets a strictly better house. Formally, a mechanism φ is *group strategy-proof* if there are no $\succeq \in \mathcal{R}^{|I|}$, $J \subseteq I$, and $\succeq'_J \in \mathcal{R}^{|J|}$ such that

$$\varphi[\succeq'_J, \succeq_{-J}](i) \succeq_i \varphi[\succeq](i) \qquad \forall i \in J, \text{ and}$$

$$\varphi[\succeq'_J, \succeq_{-J}](j) \succ_j \varphi[\succeq](j) \qquad \exists j \in J.$$

A mechanism is **(Maskin) monotonic** if whenever the preferences of agents change in a way such that the lower contour set at the assigned house under the original preferences is a subset of the lower contour set at the same house under the new preferences, then the matching assigned by the mechanism does not change (Maskin, 1999). Formally, a mechanism φ is *monotonic* if for every $\succeq, \succeq' \in \mathcal{R}^{|I|}$ and $i \in I$,

$$\{h \in H : \varphi[\succeq](i) \succeq_i h\} \subseteq \{h \in H : \varphi[\succeq](i) \succeq_i' h\} \quad \Rightarrow \quad \varphi[\succeq'] = \varphi[\succeq].$$

In this case, we say that $\succeq'$ is a **monotonic transformation** of $\succeq$ under φ.

Axioms of strategy-proofness, non-bossiness, group strategy-proofness and monotonicity are very related concepts, and the following lemmata show their relationships:

Lemma 2 *(Pápai, 2000) A mechanism is group strategy-proof if and only if it is strategy-proof and non-bossy.*

Lemma 3 *(Takemiya, 2001) A mechanism is monotonic if and only if it is group strategy-proof.*

These lemmata were previously proven in a domain without outside options but the proofs carry over to our setting. For independent interest, we provide a simple alternative proof of Lemma 2 here:

Proof of Lemma 2. One direction is obvious. To prove the other direction, suppose φ is non-bossy and strategy-proof, and, contrary to the claim, suppose it is not group strategy-proof. Then, there exists a coalition $J \subseteq I$ such that for some $\succeq$ and $\succeq'_J$, we have $\varphi[\succeq'_J, \succeq_{-J}](j) \succeq_j \varphi[\succeq](j)$ for every $j \in J$ and at least one agent has strict preference. Let $\succeq' = [\succeq'_J, \succeq_{-J}]$.

Let $\mu = \varphi[\succ]$ and $\mu' = \varphi[\succeq'_J, \succeq_{-J}]$. Consider $\succeq^*_J$ such that $\succeq^*_j$ ranks $\mu'(j)$ first and $\mu(j)$ second for every $j \in J$ such that $\mu(j) \neq \mu'(j)$ and $\succeq^*_j$ ranks $\mu(j)$ first for every $j \in J$ such that $\mu'(j) = \mu(j)$. For any $j \in J$, strategy-proofness implies that $\varphi[\succeq^*_j, \succeq_{-j}](j) \succeq^*_j \varphi[\succeq](j) = \mu(j)$, and, hence, $\varphi[\succeq^*_j, \succeq_{-j}](j) \in \{\mu(j), \mu'(j)\}$; the strategy-proofness also implies that $\mu(j) = \varphi[\succeq](j) \succeq_j \varphi[\succeq^*_j, \succeq_{-j}](j)$, and, hence, $\varphi[\succeq^*_j, \succeq_{-j}](j) = \mu(j) = \varphi[\succeq](j)$. By non-bossiness, $\varphi[\succeq^*_j, \succeq_{-j}] = \varphi[\succeq]$. Proceeding in this way, we can replace $\succeq_j$ with $\succeq^*_j$ in $\succeq$ one at a time for each $j \in J$ and we end up obtaining $\varphi[\succeq^*_J, \succeq_{-J}] = \varphi[\succeq^*_K, \succeq_{-K}] = \mu$ for every $K \subseteq J$. Proceeding similarly, we can show that $\varphi[\succeq^*_J, \succeq_{-J}] = \mu'$. Indeed, for any $j \in J$, the strategy-proofness implies that $\varphi[\succeq^*_j, \succeq'_{-j}](j) \succeq^*_j \varphi[\succeq'](j) = \mu'(j)$, and, hence, $\varphi[\succeq^*_j, \succeq_{-j}](j) = \mu'(j)$. By non-bossiness, $\varphi[\succeq^*_j, \succeq'_{-j}] = \varphi[\succeq']$. We can thus modify each $\succeq'_j$ without changing the relative ranking of $\mu(j)$ and $\mu'(j)$ by pushing all other options below $\mu(j)$ for every agent in J, one agent at a time, and we still have μ' as the outcome of φ at each step.

We have thus shown that $\varphi[\succeq'_J, \succeq_{-J}](j) = \mu' = \mu = \varphi[\succeq](j)$, a contradiction proving that φ is group strategy-proof. ■

The last concept we use is neutrality. In order to introduce it, we first define two auxiliary concepts. A **relabeling** is a function $\pi : H \cup \{\emptyset\} \to H \cup \{\emptyset\}$ is a one-to-one and onto function with $\pi(\emptyset) = \emptyset$. That is, under a relabeling, the names of houses are exchanged. Let Π be the set of relabeling functions. For example, under relabeling $\pi \in \Pi$, for house $h \in H$, $\pi(h)$ is house h's new name. For any $\succeq \in \mathcal{R}^{|I|}$, and $\pi \in \Pi$, the **relabeled preference profile** $\succeq^\pi \in \mathcal{R}^{|I|}$ is such that for any $i \in I$,

$$x \succeq^\pi_i y \quad \Longleftrightarrow \quad \pi^{-1}(x) \succeq_i \pi^{-1}(y) \qquad \forall x, y \in H \cup \{\emptyset\}.$$

That is, under the relabeled preference profile, the original names of the houses are replaced by their new names.

A mechanism is **neutral** if renaming of houses results with everybody receiving the house which is the renamed version of her old assignment. Formally a mechanism φ is *neutral* if for any $\succeq \in \mathcal{R}^{|I|}$ and $\pi \in \Pi$,

$$\varphi[\succeq^\pi](i) = \pi(\varphi[\succeq](i)) \qquad \forall i \in I.$$

3 Binary Serial Dictatorships

We now construct the class of mechanisms that characterize the axioms group strategy-proofness, neutrality, individual rationality, and non-wastefulness. We start with the definition of a standard sequential dictatorship. A **sequential order** is a function $f : \overline{\mathcal{M}} \to I$ such that $f(\sigma) \in I \setminus I^\sigma$ for every $\sigma \in \overline{\mathcal{M}}$. A **sequential dictatorship** is a mechanism ϕ^f, which is induced by a sequential order f, and its outcome is found by the following iterative algorithm given a preference profile $\succeq$:

> **Step** 1: Agent $i_1 = f(\varnothing)$ is assigned her favorite option in $H \cup \{\emptyset\}$; let this option be denoted as x_1.
>
> $\vdots$
>
> **Step** ℓ: Let $\sigma_{\ell-1} = \{(i_1, x_1), (i_2, x_2), ..., (i_{\ell-1}, x_{\ell-1})\}$. Agent $i_\ell = f(\sigma_{\ell-1})$ is assigned her favorite option in $[H \setminus \{x_1, x_2, ..., x_{\ell-1}\}] \cup \{\emptyset\}$; let this option be denoted as x_ℓ.

Sequential dictatorships are strategy-proof, non-bossy, and Pareto efficient. But they are not neutral in general. We need to restrict the set of sequential dictatorships considerably to obtain an neutral mechanism.

A **binary serial order** is a sequential order f such that for every $\sigma, \sigma' \in \overline{\mathcal{M}}$ satisfying $I^\sigma = I^{\sigma'}$ and $\sigma(i) = \emptyset \iff \sigma'(i) = \emptyset$ for every $i \in I^\sigma$, we have $f(\sigma) = f(\sigma')$. We refer to a sequential dictatorship induced by a binary serial order as a **binary serial dictatorship**.

Because the next agent with the priority to choose is fully determined by who among the previous agents are assigned the outside option, we can simplify our notation for binary serial order. Let $\mathcal{B} = \{J \times \{0, 1\}\}_{J \subsetneq I}$. An element $\beta \in \mathcal{B}$ is referred to as a **binary submatching** and can also be denoted in the functional form as well, i.e., $\beta(i) = b$ whenever $(i, b) \in \beta$. Moreover, let $(i, 0)$ refer to "i is assigned the outside option" and $(i, 1)$ refer to "i is assigned a house." Let $I^\beta = J$ be the set of agents matched under β. Redefine a binary serial order as a function $f : \mathcal{B} \to I$ such that for every $\beta \in \mathcal{B}$, $f(\beta) \in I \setminus I^\beta$. Moreover, we refer to $\sigma \in \mathcal{S}$ as **consistent with** $\beta \in \mathcal{B}$ if $I^\sigma = I^\beta$ and $\beta(i) = \mathbb{I}\{\sigma(i) \in H\}$ for every $i \in I^\beta$.[6]

Given a sequential order f, a **relevant submatching** is a submatching $\sigma = \{(i_k, x_k)\}_{k=1}^{\ell}$ for some $\ell \geq 0$ such that $i_k = f(\{(i_1, x_1), ..., (i_{k-1}, x_{k-1})\})$ for all $k < \ell$. Let $\mathcal{S}^f$ be the set of all relevant submatchings for f. These definitions hold for binary submatchings, too. In particular, let $\mathcal{B}^f$ be the set of **relevant binary submatchings** for a binary serial order f.

Serial dictatorships are a subclass of binary serial dictatorships (and hence of sequential dictatorships): a **serial dictatorship** is a sequential dictatorship ϕ^f such that $f(\sigma) = f(\sigma')$ for every $\sigma, \sigma' \in \overline{\mathcal{M}}$ such that $I^\sigma = I^{\sigma'}$. We refer to such a sequential order f as a **linear order**.

4 The Characterization

Our main result is as follows:

Theorem 1 *A mechanism is group strategy-proof, neutral, individually rational, and non-wasteful if and only if it is a binary serial dictatorship.*

Proof of Theorem 1.

$\Longleftarrow$ Let ϕ^f be a binary serial dictatorship. Then, ϕ^f is a hierarchical exchange mechanism; these class of mechanisms was introduced by Pápai (2000) for settings without outside options and extended to the setting with outside options by Pycia and Ünver (2011). Pápai (2000) showed that in her setting hierarchical exchange mechanisms are group strategy-proof and Pareto efficient, and this insight as well as Pápai's proof extend to our setting; see Pycia and Ünver's work for more details. Thus, ϕ^f is group strategy-proof and Pareto efficient; Pareto efficiency implies non-wastefulness and individual rationality. Because the definition of ϕ^f does not depend on the names of houses assigned, ϕ^f is also neutral.

[6] $\mathbb{I}\{\kappa\} = 1$ if κ is a true statement, and $\mathbb{I}\{\kappa\} = 0$ otherwise.

$\Longrightarrow$ Let φ be a group strategy-proof, neutral, non-wasteful, and individually-rational mechanism. By Lemma 1, φ is individually rational. By Lemma 2, φ is strategy-proof and non-bossy. By Lemma 3, φ is monotonic.

We introduce two definitions for the proof:[7]

Definition. An *ordered submatching* is defined as a list $\sigma = \big((i_1, y_1), \ldots, (i_\ell, y_\ell)\big)$ such that $\{(i_1, y_1), \ldots, (i_\ell, y_\ell)\} \in \mathcal{S}$. With a slight abuse of notation, suppose σ also refers to the corresponding submatching $\{(i_1, y_1), \ldots, (i_\ell, y_\ell)\}$ whenever needed.

Definition. Given an ordered submatching $\sigma = \big((i_1, y_1), \ldots, (i_\ell, y_\ell)\big)$, let $\mathcal{R}^{\sigma;x_1,\ldots,x_k}$ be the domain of the following preference profiles $\succeq$ with distinct options $x_1, \ldots, x_k \in \big(H \setminus H^\sigma\big) \cup \{\emptyset\}$:

1. for every $i_m \in I^\beta$, $\succeq_{i_m}$ ranks the houses $\{y_1, y_2, ..., y_{m-1}\} \cap H$ in order of their indices, then y_m, and then $\{y_{m+1}, y_{m+2}, ..., y_\ell\} \cap H$ in order of their indices, and then $\{x_1, x_2, ..., x_k\}$ in order of their indices, and finally other options in arbitrary order.
2. for every $i \in I \setminus I^\beta$, $\succeq_i$ ranks houses $\{y_1, y_2, ..., y_\ell\} \cap H$ in order of indices, then $x_1, \ldots, x_k$, in this order and then other options.

We iteratively construct the binary serial order f and prove the claim showing the order is well defined.

Step 1: Fix a house $h_\varnothing \in H$. Fix a profile $\succeq^\varnothing \in \mathcal{R}^{\varnothing;h_\varnothing,\emptyset}$. $\varphi^{-1}[\succeq^\varnothing](h) \in I$ by non-wastefulness of φ. Let $f(\varnothing) = \varphi^{-1}[\succeq^\varnothing](h_\varnothing)$.

$\vdots$

Step ℓ: For every $\beta \in \mathcal{B}$ with $|I^\beta| = \ell - 1$ we do the following:

1. If $\beta \notin \mathcal{B}^f$, then pick $f(\beta) \in I \setminus I^\beta$ arbitrarily.
2. Otherwise, let $\beta' \subset \beta$ be the unique relevant binary submatching of β with $|I^{\beta'}| = \ell - 2$. Fix an ordered submatching $\sigma_\beta = \big(\sigma_{\beta'}, (j_{\ell-1}, y_{\ell-1})\big)$ where $j_{\ell-1} = f(\beta')$ as constructed in Step $\ell - 1$ and let $y_{\ell-1}$ be such that its choice makes σ_β consistent with β, i.e., $y_{\ell-1}$ is a house if $\beta(j_{\ell-1}) = 1$ and $y_{\ell-1} = \emptyset$ if $\beta(j_{\ell-1}) = 0$.
   (a) If $H \setminus H^{\sigma_\beta} = \emptyset$, then pick $f(\beta) \in I \setminus I^\beta$ arbitrarily.
   (b) Otherwise, fix a house $h_\beta \in H \setminus H^{\sigma_\beta}$ (and in particular if $H \setminus H^{\sigma_\beta} = H \setminus H^{\sigma_{\beta'}}$, then pick $h_\beta = h_{\beta'}$) and a preference profile $\succeq^\beta \in \mathcal{R}^{\sigma_\beta;h_\beta,\emptyset}$. By non-wastefulness of φ, $\varphi^{-1}[\succeq^\beta](h_\beta) \in I$.
   Let $f(\beta) = \varphi^{-1}[\succeq^\beta](h_\beta)$.

Claim. *f is a well-defined binary serial order. Moreover, for every $\beta \in \mathcal{B}^f$ such that $|I^\beta| = \ell \in \{0, \ldots, |I| - 1\}$ and ordered submatching $\sigma = \big((j_1, x_1), \ldots, (j_\ell, x_\ell)\big)$ which is consistent with β and*

[7]For ease of reading, we display important newly defined objects and terms in display in the proof.

satisfies $\{(j_1,x_1),\ldots,(j_k,x_k)\}\in\mathcal{S}^f$ *for every* $k\le\ell$, *we have that if* $H\setminus\{x_1,x_2,...,x_\ell\}\neq\emptyset$, *then for every* $h\in H\setminus\{x_1,x_2,...,x_\ell\}$ *and* $\succeq\in\mathcal{R}^{\sigma;h,\emptyset}\cup\mathcal{R}^{\sigma;\emptyset}$

$$\varphi[\succeq](j_k)=x_k=\phi^f[\succeq](j_k)\qquad\forall k\le\ell\qquad \textit{and} \tag{1}$$

$$\varphi[\succeq](f(\beta))=\begin{cases} h & \textit{if } \succeq\in\mathcal{R}^{\sigma;h,\emptyset}\\ \emptyset & \textit{otherwise}\end{cases}. \tag{2}$$

Proof of Claim. We prove the claim by induction on $\ell=\left|I^\beta\right|$.

Step 1: We have $\beta=\varnothing$ and thus, any consistent submatching is $\varnothing$. By construction of f in Step 0, $\varphi[\succeq^\varnothing](f(\varnothing))=h_\varnothing$. By individual rationality of φ, we have $\varphi[\succeq^\varnothing](i)=\emptyset$ for every $i\in I\setminus\{f(\varnothing)\}$. Therefore, every $\succeq\in\mathcal{R}^{\varnothing;h_\varnothing,\emptyset}$ is a monotonic transformation of $\succeq^\varnothing$ under φ. This, in turn, together with the monotonicity of φ imply that $\varphi[\succeq]=\varphi[\succeq^\varnothing]$ and $\varphi[\succeq](f(\beta))=h_\varnothing$. Moreover, by neutrality of φ, for every $h\in H$ (by using a relabeling that renames $h_\varnothing$ as h and keeps the names of the houses as they are), and $\succeq'\in\mathcal{R}^{\varnothing;h,\emptyset}$ we have $\varphi[\succeq'](f(\beta))=h$. By individual rationality, for all $\succeq''\in\mathcal{R}^{\varnothing;\emptyset}$ we have $\varphi[\succeq''](f(\beta))=\emptyset$.

Step $\ell+1$: As our inductive assumption, suppose the claim holds for any $\beta''\in\mathcal{B}^f$ with $|I^{\beta''}|<\ell$ and f is well defined for all relevant binary submatchings $\beta'\subseteq\beta''$. Fix $\beta\in\mathcal{B}^f$ with $|I^\beta|=\ell$. If $|I^\beta|\ge|H|$, then the induction was proved in previous steps. It remains to consider $|I^\beta|<|H|$.

Let $\beta'\subset\beta$ be the unique relevant submatching of β with $|I^{\beta'}|=\ell-1$. We show that for any $J\subseteq I\setminus I^\beta$ and $k\le\ell$,

$$\varphi[\succeq^\beta_{I^\beta\cup J},\succeq^{\beta'}_{-I^\beta\cup J}](j_k)=y_k=\phi^f[\succeq^\beta_{I^\beta\cup J},\succeq^{\beta'}_{-I^\beta\cup J}](j_k) \tag{3}$$

We prove this equation by induction on $t=|J|$:

> **Step** $\ell+1.0$: For $J=\emptyset$, monotonicity of φ and inductive assumption for $t-1$ together imply Eq. (3).
>
> **Step** $\ell+1.t$ for $t>0$: Fix $J\subseteq I\setminus I^{\sigma_\beta}$ such that $|J|=t$. Suppose as the inductive assumption, for all $J'\subset J$, Eq. (3) holds. Denote:
>
> $$\hat{\succeq}=(\succeq^\beta_{I^\beta\cup J},\succeq^{\beta'}_{-I^\beta\cup J}),$$
>
> and pick $i\in J$. By individual rationality of φ and inductive assumption for $\ell-1$, we have $\varphi[\succeq^{\beta'}_i,\hat{\succeq}_{-i}](i)=\emptyset$. Then by strategy-proofness of φ, $\varphi[\hat{\succeq}](i)\in\{h_\beta,\emptyset\}$.
>
> If $\varphi[\hat{\succeq}](i)=\emptyset$, then by non-bossiness of φ, $\varphi[\hat{\succeq}]=\varphi[\succeq^{\beta'}_i,\hat{\succeq}_{-i}]$. Eq. (3) for J follows from inductive assumption for $\ell-1$. Suppose $\varphi[\hat{\succeq}](i)=h_\beta$, which is the sole remaining possibility. By way of contradiction, suppose there exists some $j_k\in I^\beta$ such that
>
> $$\varphi[\hat{\succeq}](j_k)\neq y_k.$$

We further choose j_k such that $y_k \in H$ and k is the largest index in $\{1, \dots, \ell\}$ such that above inequality and $y_k \in H$ hold.[8] By non-wastefulness of φ, there exists some agent $j \notin \{i, j_k\}$ such that[9]

$$\varphi[\hat{\succeq}](j) = y_k.$$

Notation Definition: Let houses assigned in σ_β to agents $j_1, \dots, j_{k-1}$ be

$$h_1, \dots, h_{p-1}$$

in this order and

$$h_p := y_k$$

(and thus, $\{y_1, \dots, y_k\} \cap H = \{h_1, \dots, h_p\}$). Let the houses assigned in σ_β to agents $j_{k+1}, \dots, j_t$ be

$$h_p + 1, \dots, h_q$$

in this order (and thus, $\{y_{k+1}, \dots, y_\ell\} \cap H = \{h_{p+1}, \dots, h_q\}$.[10] Finally let,

$$h_{q+1} := h_\beta.$$

Thus, with the new house notation, as a reminder, we have

$$\varphi[\hat{\succeq}](j) = h_p \quad \text{and} \quad \varphi[\hat{\succeq}](i) = h_{q+1}.$$

We make a possible modification to the definition of $\hat{\succeq}$:

- If $j \in I \setminus (I^\beta \cup J)$, then extend J to include j:

$$\hat{J} := J \cup \{j\},$$

  and modify $\hat{\succeq}_j$ as follows: Instead of h_{q+1} being unacceptable for j, let her rank h_{q+1} just below h_q and above $\emptyset$ (like the agents in J). By monotonicity of φ, the outcome of the mechanism remains the same as j receives h_p under original $\hat{\succeq}$, and it is ranked at least as high as h_q by her.
- Otherwise, let

$$\hat{J} := J.$$

[8] Such an agent j_k exists with $y_k \in H$ due to the following observation: Suppose $y_k = \emptyset$ then since $\varphi[\hat{\succeq}](j_k) \neq y_k$ agent j_k should be receiving a house $y_{\hat{k}} \in \{y_1, \dots, y_{\hat{k}-1}\} \cap H$ under $\varphi[\hat{\succeq}]$ by construction of $\succeq^\beta_{I^\beta}$. Thus, in return agent $j_{\hat{k}}$ is now receiving a different option. Therefore, we can use $j_{\hat{k}}$ instead of j_k in this case.

[9] By choosing k as large as possible above, we ensure that if $j = j_m$ for some index $m \in \{1, \dots, k-1\}$ then $y_m \hat{\succ}_{j_m} y_k$ and thus y_m is a house by individual rationality of φ, as otherwise $\varphi[\hat{\succeq}](j) = \emptyset$ would hold.

[10] To avoid any confusion, we use the subscripts k, ℓ, m, n to denote the indices of specific agents in I^β and the options that are assigned to them in σ_β (where k and ℓ are already fixed), and we use subscripts p, q, r, s to denote the indices of actual houses assigned to these agents in σ_β (where p and q are fixed in the definition now). Thus, whenever $y_m = h_r$, as some of the options assigned can be $\emptyset$ in σ_β, we have $r \leq m$ in all cases.

We define a new preference profile as follows:

Definition. Let $\tilde{\succeq}$ be a profile such that

1. $\tilde{\succeq}_j$ is obtained from $\hat{\succeq}_j$ by demoting h_p in rankings just below h_{q+1}.[11]
2. $\tilde{\succeq}_{-j}$ is obtained from $\hat{\succeq}_{-j}$ by demoting h_p in rankings so that it is unacceptable for everyone different from j (if it is already not).

Profile $(\hat{\succeq}_j, \tilde{\succeq}_{-j})$ is a monotonic transformation of $\hat{\succeq}$ under φ. By monotonicity of φ, we have

$$\varphi[\hat{\succeq}_j, \tilde{\succeq}_{-j}] = \varphi[\hat{\succeq}].$$

By strategy-proofness of φ,

$$\varphi[\tilde{\succeq}_j, \tilde{\succeq}_{-j}] \in \{h_p + 1, \ldots, h_{q+1}, h_p\}.$$

We define the following relabeling that will be used in the remainder of the proof:

Definition: Let relabeling π be such that

$$\begin{aligned} \pi(h_p) &= h_{q+1}, \\ \pi(h_r) &= h_{r-1} \qquad \forall\, h_r \in \{h_{p+1}, \ldots, h_{q+1}\}, \text{ and} \\ \pi(h) &= h \qquad \forall\, h \in H \setminus \{h_p, \ldots, h_{q+1}\}. \end{aligned}$$

We have two cases:

Case 1. $\varphi[\tilde{\succeq}](j) = h_p$: Then by non-bossiness of φ, $\varphi[\tilde{\succeq}] = \varphi[\hat{\succeq}]$. By neutrality of φ we have

$$\varphi[\tilde{\succeq}^{\pi}](i) = \pi(h_{q+1}) = h_q.$$

We have $h_q = y_n$ for some index $n \leq \ell$. Moreover, $i \neq j_n$ as $i \in \hat{J}$. Consider the relevant binary submatching $\tilde{\beta} \subset \beta$ with $|I^{\tilde{\beta}}| = n - 1$:

$$\sigma_{\tilde{\beta}} = \big((j_1, y_1), \ldots, (j_{n-1}, y_{n-1})\big).$$

Observe that $\tilde{\succeq}^{\pi}$ is a monotonic transformation of $\succeq^{\tilde{\beta}}$ under φ (used in the construction of $f(\tilde{\beta})) = j_n$ under φ in Step $n+1$ of the construction, which is well defined by inductive assumption for $n+1$). By monotonicity of φ,

$$\varphi[\tilde{\succeq}^{\pi}] = \varphi[\succeq^{\tilde{\beta}}].$$

However, this contradicts $\varphi[\succeq^{\tilde{\beta}}](j_n) = y_n = h_q$ as we have $\varphi[\tilde{\succeq}^{\pi}](i) = y_n = h_q$.

[11] As explained in Footnote 9, if $j = j_m \in I^{\beta}$, we have $y_m \neq \emptyset$; and therefore, house h_{q+1} is an acceptable choice for j by construction of the preference profile subdomain $\mathcal{R}^{\sigma_\beta; h_{q+1}, \emptyset}$. Hence, h_p remains as an acceptable choice under $\tilde{\succeq}_j$.

Case 2. $\varphi[\tilde{\succeq}](j) = h_s \in \{h_{p+1}, \ldots, h_q\} = \{y_{k+1}, \ldots, y_\ell\} \cap H$. Now, $h_s = y_n$ for some $n \in \{k+1, \ldots, \ell\}$. By choice of j, $j_n \neq j$.[12] Modify $\tilde{\succeq}_j$ further so that h_p is ranked below $\emptyset$, and otherwise the rankings of options are unchanged. Observe that this change does not change $\varphi[\tilde{\succeq}]$ by φ's monotonicity. By neutrality of φ we have

$$\varphi[\tilde{\succeq}^\pi](j) = \pi(h_s) = h_{s-1}.$$

Let the relevant binary submatching $\tilde{\beta} \subset \beta$ be such that

$$\sigma_{\tilde{\beta}} = \big((j_1, y_1), \ldots, (j_{\ell-1}, y_{\ell-1})\big).$$

Observe that $\tilde{\succeq}^\pi$ is a monotonic transformation of $\succeq^{\tilde{\beta}}$ under φ (used in the construction of $f(\tilde{\beta})) = j_n$ and well defined by the inductive assumption for Step $n+1$). By monotonicity of φ,

$$\varphi[\tilde{\succeq}^\pi] = \varphi[\succeq^{\tilde{\beta}}].$$

However, this contradicts, $\varphi[\succeq^{\tilde{\beta}}](j_n) = y_n = h_s$ as we have $\varphi[\tilde{\succeq}^\pi](j) = y_n = h_s$.

In either case, we found that such an agent j cannot exist proving the inductive statement Eq. (3)'s first equality for J. The second equality of Eq. (3) for J follows from the construction of f up to Step ℓ by the outer inductive assumption for Step ℓ, and definition of the binary serial dictatorship ϕ^f .

Thus, f is well defined in Step $\ell+1$. By neutrality of φ, Eq. (1) for Step $\ell+1$, i.e., the first part of the outer inductive statement for Step $\ell+1$, follows. Eq. (2) for ℓ, i.e., the second part of the outer inductive statement for Step $\ell+1$, follows from neutrality of φ and construction of f if $\succeq \in \mathcal{R}^{\sigma_\beta;h,\emptyset}$ and by individual rationality of φ if $\succeq \in \mathcal{R}^{\sigma_\beta;\emptyset}$. This ends the proof of the claim. $\diamondsuit$

To finish the proof of the theorem, take an arbitrary $\succeq \in \mathcal{R}^{|I|}$. Let $\mu = \phi^f[\succeq]$. Let $\beta \in \mathcal{B}^f$ such that $|I^\beta| = |I| - 1$, and let σ be an ordered submatching that is consistent with β such that

$$\sigma = \big((j_1, \mu(j_1), \ldots, (j_{|I|-1}, \mu(j_{|I|-1})\big)$$

and

$$\big((j_1, \mu(j_1), \ldots, (j_k, \mu(j_k))\big) \in \mathcal{S}^f \qquad \forall\, k \leq |I| - 1.$$

Let $\{j_{|I|}\} = I \setminus I^\beta$ be the singleton including the last remaining person in I^β. If $\mu(j_{|I|}) \in H$, then let $\succeq' \in \mathcal{R}^{\sigma;\mu(j_{|I|}),\emptyset}$ and otherwise, let $\succeq' \in \mathcal{R}^{\sigma;\emptyset}$. By the above Claim, we have

$$\varphi[\succeq'] = \phi^f[\succeq'].$$

Therefore, $\succeq$ is a monotonic transformation of $\succeq'$ under both φ and ϕ^f, which are both monotonic. Then

$$\varphi[\succeq] = \varphi[\succeq'] = \phi^f[\succeq'] = \phi^f[\succeq].$$

■

[12] As either $j \notin I^\beta$ or if $j = j_m \in I^\beta$ then $m \in \{1, \ldots, k-1\}$ by k being the largest index possible; see Footnote 9.

5 Independence of the Axioms

By relaxing each axiom one at a time, we now show that there exists a mechanism which is not a binary serial dictatorship and yet satisfies the remaining axioms. We also show that the axioms remain independent if we substitute strategy-proofness and non-bossiness for group strategy-proofness (see Lemma 1 for this two-axiom reformulation of grup strategy-proofness).

Example 1 A mechanism that is **non-strategy-proof**, non-bossy, neutral, non-wasteful, and individually rational: Take a binary serial order. Run the associated binary serial dictatorship with the following modification: Reverse the preference order of each agent for all houses that she ranked higher than the outside option and keep the relative order of other options the same.

Example 2 A mechanism that is strategy-proof, **bossy**, neutral, non-wasteful, and individually rational: Let f, f' be two binary serial orders such that $f(\varnothing) = f'(\varnothing) = i \in I$, but otherwise the orders do not match in general, i.e., $f \neq f'$. Let φ be a mechanism such that

$$\varphi[\succeq] = \begin{cases} \phi^{f}[\succeq] & \text{if } h \succ_i \emptyset \quad \forall h \in H \\ \phi^{f'}[\succeq] & \text{otherwise} \end{cases},$$

i.e., the binary serial order that will be used in the binary serial dictatorship is determined by the preferences of the initial *dictator* (but not necessarily by her assigned option), depending on whether she prefers all houses to the outside option or not.

Example 3 A mechanism that is strategy-proof, non-bossy, **non-neutral**, non-wasteful, and individually rational: A top-trading-cycles mechanism (a la Pápai, 2000) that gives the ownership rights of objects to at least two different agents at the beginning.

Example 4 A mechanism that is strategy-proof, non-bossy, neutral, **wasteful**, and individually rational: A mechanism that leaves every agent always unmatched.

Example 5 A mechanism that is strategy-proof, non-bossy, neutral, non-wasteful, and **individually irrational**: Take a binary serial order. Run the associated binary serial dictatorship with the following modification: During her turn each agent is assigned the best available house according to her preferences if there are still available houses (even if the outside option is preferred to that house) and the outside option otherwise.

6 Serial Dictatorships with Outside Options

As an application of our binary serial dictatorship characterization, we extend Svensson's serial dictatorship characterization result to the subdomain of our preference domain in which the outside option is always ranked at the bottom of preferences. This is the preference domain analyzed in Pápai (2000) and in the following literature. Let this restricted set of preferences be denoted by $\hat{\mathcal{R}}$.

Theorem 2 *A mechanism defined over $\hat{\mathcal{R}}^{|I|}$ is group strategy-proof, neutral, and non-wasteful if and only if it is a serial dictatorship.*

The proof follows verbatim the proof of Theorem 1 using the restricted domain instead of the larger domain.[13] Note that—unlike in Svensson's original characterization—non-wastefulness is not a redundant axiom: a mechanism that leaves all agents unmatched satisfies all axioms but non-wastefulness.

7 Conclusion: Scarcity vs Abundance

A natural reading of our result is that in the presence of incentive and efficiency assumptions, neutral mechanisms belong to the class of sequential dictatorships. How general is this insight? How different is our version of it from Svensson's (1999) characterization of serial dictatorships without outside options? To get a sense of the difference between our result and Svensson's, consider a setting with houses and cars, with no outside options. We can embed Svensson's setting in this environment by assuming that there are no cars and we can embed our setting in this environment by assuming that cars are abundant. In this general environment, let us consider mechanisms that are neutral with respect to houses. This concept reduces to neutrality in both Svensson's and our setting. In this environment, the following mechanisms are neutral: any **top–trading–cycles mechanisms** (TTC, hierarchical exchange) of Pápai (2000) in which in each round one agent controls houses and a different agent controls cars. Each of these two agents would get her top available choice if it is controlled by her, and they would trade with each other if they like each other's objects the most. However, whenever there is a conflict in their top choices, the agent holding the property right for the object they both want would receive the object. With three or more types of objects (e.g. houses, cars, and boats), we can even have **trading–cycles mechanisms** (TC) of Pycia and Ünver (2017) that are neutral among houses. Similar mechanism classes arise in settings when some objects have copies as long as the number of copies is strictly less than $|I|$. Neutrality among houses leads to sequential dictatorships when the goods are very scarce or when they are abundant, but in general, subclasses of sequential dictatorships cannot characterize neutral, group–strategy-proof, non-wasteful, and individually-rational mechanisms. Instead, the environment we study and the one studied by Svensson are two extreme cases of the general environment in which the connection between sequential dictatorships and neutrality is true.[14]

On the other hand, analogues of our results can be derived for domains with multiple abundant goods. Suppose for instance that we have multiple types of outside options that can each be attained by any agent, and agents value such outside options differently. In this case, a variant of our characterization would continue to hold: In this case, instead of a binary serial dictatorship, we will obtain a $(\ell + 1)$–*sequential dictatorship* where ℓ is the number of different outside options, and recursively define its sequential order f as follows: for each relevant ordered submatching

[13]Since the preference domain of this result is included in Pycia and Ünver (2017), this proposition can alternatively be proven using their characterization. However, Theorem 1 does not as readily follow from their results.

[14]Not surprisingly, Svensson's proof approach fails in our setting.

$\sigma = \big((i_1, x_1), (i_2, x_2), \ldots, (i_{n-1}, x_{n-1})\big)$, once agent $f(\sigma)$ is defined as the next agent who owns all available houses, for any ordered submatching $\sigma' = \big(\sigma, (f(\sigma), x_n)\big)$ agent $f(\sigma')$ can be $\ell + 1$ different agents depending on x_n being one the ℓ outside options or a real house. Our proof can be extended to this case.

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